

PH142 2019 Extra Practice SOLUTIONS

1. Which of the following statements is **INCORRECT** for the F distribution?

- a) The F distribution is skewed to the right
- b) It can only take positive values
- c) The p-value of F statistics is always the area to the right of the test statistic
- d) None of the above

SOLUTION: d) None of the above

2. Fill in the blank: _____ is a hypothesis test that reshuffles the data to break any relationship between two variables. The null hypothesis of this test is that there is no difference between the two sampling distributions.

SOLUTION: permutation tests

3. Circle one term for each blank: For non-parametric methods, _____ [bootstrap/permutation test] is for hypothesis testing, whereas _____ [bootstrap/permutation test] is for confidence intervals.

SOLUTION: permutation test; bootstrap

4. [1 point] True or false: The chi-squared distribution is not symmetric.

SOLUTION: True

5. True or false: Under the null hypothesis for the unpaired two-sample t-test of the means for populations with unequal variances, the degrees of freedom for the test statistic is equal to $n_1 + n_2 - 2$, where n_1 and n_2 are the sample sizes of the samples.

SOLUTION: False

6. True or false: A test for comparing two proportions can also be conducted using a Chi-square test.

SOLUTION: True. In a chi-square test, we can compare expected vs. observed proportions for groups.

7. True or false: If a 90% confidence interval for a proportion is (0.442, 0.542), then the margin of error is 0.10.

SOLUTION: False

8. True or False: Running a one proportion test means I calculate a one-sided p-value, while running a two proportion test means I calculate a two-sided p-value.

SOLUTION: False. One-sided and two-sided p-values are specified by the alternative hypothesis. If the alternative hypothesis test is that the proportion is NOT equal to the null hypothesized proportion, then it is two-sided. If it is greater than, then it would be a one-sided, same with less than.

Question 9

Sleep deprivation specialists are interested in determining whether the number of hours a student sleeps per night depends on whether they have an upcoming statistics exam. The researchers posit that the stress of the exam will not influence the students' sleep patterns, but the students disagree. Twelve randomly selected students were asked to record the number of hours they slept on the night two weeks prior to the day of the exam. The same students were also asked to record the number of hours slept the night before the exam. The data is presented in the table below.

Table 1: Number of hours slept

	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11
Two weeks before exam	10.3	10.7	9.9	8.4	8.8	10.1	9.7	9	8	9.2	10.5
Night before exam	5.3	3.9	4.5	4.3	4.4	4.1	2.6	4.6	2.4	2.9	3.6

Assume that all assumptions required for the use of the appropriate procedure to make a confidence interval are met.

- a) State the null and alternative hypotheses.

SOLUTION:

$$H_0 : \mu_d = 0$$

$$H_A : \mu_d \neq 0$$

Where d is the difference between the number of hours slept two weeks before the exam and the night before exam

- b) Use the appropriate t-procedure to compute the 95% confidence interval of the difference between the number of hours slept two weeks before the exam and the night of the exam. Interpret the confidence interval.

You'll need to select one of the following critical values to compute the confidence interval:

- a) $qt(0.95, df = 11) = 1.80$
- b) $qt(0.025, df = 11) = -2.20$
- c) $qt(0.975, df = 12) = 2.18$
- d) $qt(0.975, df = 22) = 2.07$

You'll also need the following value: $s = 1.43$

SOLUTION:

Use the matched pairs t-test. The mean is 5.35. The standard deviation is 1.43 and the the correct critical value is given by the negative value of b). Thus, the confidence interval is:

$$(5.35 - 2.20 * 1.43/\sqrt{(12)}, 5.35 + 2.20 * 1.43/\sqrt{(12)}) = (4.44, 6.26)$$

Under repeated sampling, we would expect to find the true mean difference between number of hours slept two weeks before and exam and the night before the exam to be in the 95% CI of the samples 95% of the time.

Question 10

Suppose that PH142 has five discussion sections. We want to know whether they have the same exam performance. We select a random sample of students in each of these sections, and run an ANOVA analysis.

- a) State the null and alternative hypotheses.

SOLUTION:

$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$ or the 5 sections have same exam performance or same mean.

H_A : Not all $\mu_1 \dots \mu_5$ are the equal, or at least one of the means differ.

- b) Use information in the provided table cells in addition to other information from the question to complete the ANOVA table.

Term	df	sumsq	meansq	statistic	p.value
Sections	A	1.2	0.3	C	0.01159
Residuals	195	15.6	B	NA	NA

SOLUTION:

A: 4

B: $15.6/195 = 0.08$

C: $0.3/0.08 = 3.75$

- c) How many total students were in the sample?

$$N = 195 + 4 + 1 = 200$$

- d) Interpret the p-value in the context of this problem.

SOLUTION:

The p value is 1.16%, which means under the null hypothesis, we have 1.16% chance of observing the F statistic we calculated or a more extreme value. This is a very small probability and provides evidence against the null hypothesis that all sections have the same performance in favor of the alternative that at least one section has different average performance.

Question 11

Suppose you were presented with the following results after running a linear regression using two continuous variables. Which number can be used to quantify a probability having to do with the correlation between freq and temp? State the hypotheses and make a conclusion based on the number you mentioned above.

```
## # A tibble: 2 x 5
##   term          estimate std.error statistic    p.value
##   <chr>          <dbl>    <dbl>    <dbl>    <dbl>
## 1 (Intercept)   -6.19     8.24    -0.751  0.462
## 2 temp          2.33     0.347    6.72  0.00000266
```

SOLUTION:

(Recall that the test of the slope term is equivalent to test of correlation). The number being used is the p-value in the temp row. The corresponding null hypothesis is that there is no correlation between temp and freq (alternatively, could say the null is that slope term $b = 0$); Alternative hypothesis is there is a correlation between temp and freq (or could say $b \neq 0$). Since the p-value is very small ($<0.01\%$), we will reject the null hypothesis and find evidence in favor of a correlation between these variables.

Question 12

The approximate frequencies of ABO blood types among all Americans are given in the table below.

O	A	B	AB
44%	42%	10%	4%

While determining how much of each blood type to keep in stock, a particular hospital investigates whether the distribution of blood types among its patients is the same as the distribution of blood types nation-wide. The investigators collect a simple random sample of 250 patients seen at the hospital over the last year. The investigators' results are shown in the table below.

O	A	B	AB	Total
90	110	40	10	250

The investigators perform a χ^2 goodness-of-fit test to determine whether the distribution of blood types among the patients at this hospital differ from the distribution of blood types among Americans in general using a significance level of 0.05.

- a) Clearly state the hypotheses and calculate the degrees of freedom and test statistic.

SOLUTION:

H_0 : The proportion of O, A, B, and AB blood types among patients at this hospital is 44%, 42%, 10%, and 4%, respectively.

H_A : At least one of the proportions in the null hypothesis is false.

Expected counts:

O	A	B	AB	Total
110	105	25	10	250

$$df = k - 1 = 4 - 1 = 3$$

$$\text{Test statistic: } \chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

$$\chi^2 = \frac{(90-110)^2}{110} + \frac{(110-105)^2}{105} + \frac{(40-25)^2}{25} + \frac{(10-10)^2}{10}$$

$$\chi^2 = \frac{400}{110} + \frac{25}{105} + \frac{225}{25} + \frac{0}{10}$$

$$\chi^2 = 3.636364 + 0.2380952 + 9 + 0 = 12.87446$$

- b) Write the R code you would need to find the p-value for this test.

SOLUTION:

```
pchisq(12.87446, df = 3, lower.tail = F)
```

- c) Suppose that the p-value provides strong evidence against the null in favor of the alternative hypothesis. Which of the four blood type categories contributed the most evidence to this difference?

SOLUTION: This p-value means that there is a 0.49% chance of observing this chi-square statistic or a larger one under the null hypothesis. This chance is small enough that we reject the null hypothesis that the distribution of ABO blood types among patients at this hospital is equivalent to that for the general US population, and have evidence supporting the alternative hypothesis that at least one of the blood type proportions is different